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of his doctorate and in honor of his 70th birthday], 49–56; "Der mathematische Verein München" by H. Wieleitner, 56–58; "Emil Lampe zum Gedächtnis" [with portrait] by E. Haentzschel, 58–60; "Bücherbesprechungen," "Zeitschriftenschau," and "Neuerscheinungen," 62–64—No. 2, published Feb. 20: "Einteilung der Dreiflache" by R. Sturm, 65–66; "Ueber Winkelteilung" by R. Sturm, 66–69; "Versuch einer Bestimmung der mittleren Strafzeit" by R. Sturm, 70–71; "Zum geometrischen Anfangsunterricht" by G. Kewitsch, 71–78; "Winkel an den Parallelen" by Münch, 78–81; "Kleine Mitteilungen," 81–93; "Aufgaben-Repertorium," 93–97; "Bücherbesprechungen," 100–112—No. 3, published March 21: "Ueber das 'Linearzeichnen' an den höheren Schulen und sein Verhältnis zum 'Freihandzeichnen'" by F. Schilling, 113–131; "Die Kollineation in Prima" by A. Gerlach, 131–137; "Kleine Mitteilungen," "Bücherbesprechungen," "Zeitschriftenschau," and "Neuerscheinungen," 138–152.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

2793. Proposed by J. L. RILEY, Stephenville, Texas.

If a, b, and c are complex and α, β, and γ real constants, the point

$$x = \frac{at^2 + 2bt + c}{\alpha t^2 + 2\beta t + \gamma}$$

traces a conic or a straight line when t takes all real values. *Stolz und Gmeiner.*

2794. Proposed by B. J. BROWN, Kansas City, Mo.

Find the value of $x^{e^x} \div x^{x^x}$ when $x \doteq 0$ and when $x \doteq \infty$. *I. C. S.* 1902.

2795. Proposed by C. N. SCHMALL, New York City.

A square is described touching the ellipse $x^2/a^2 + y^2/b^2 = 1$, at the ends of its minor axis; a second ellipse is drawn circumscribing the square and tangent to the given ellipse at the ends of the major axis. The new ellipse is treated as the first and the process is continued until there are n new ellipses. Show that the last ellipse is a circle if the eccentricity of the original ellipse is $\sqrt{n/(n+1)}$.

2796. Proposed by N. P. PANDYA, Amreli, India.

Construct a triangle ABC having its centroid on a given ellipse, AB being a fixed diameter of the ellipse and C lying on one of the directrices.

2797. Proposed by E. J. OGLESBY, College of William and Mary.

Solve for x and y the simultaneous equations, $x^3 + y^3 = 35$ and $x^2 + y^2 = 13$.

2798. Proposed by S. A. COREY, Des Moines, Iowa.

Prove that every positive integer is equal to the sum of at most four squares.

SOLUTIONS OF PROBLEMS.

406 (Algebra) [March, 1914]. Proposed by S. A. COREY, Albia, Iowa.

Solve the system of equations:

$$(1 - x)(a_1 + a_2 y + a_3 z) = d, \qquad (1 - y)(b_1 + b_2 x + b_3 z) = g, \qquad (1 - z)(c_1 + c_2 x + c_3 y) = h.$$

Discussion by H. S. Uhler, Yale University.

In order to introduce cyclic order we shall interchange two of the coefficients in the second equation and rewrite it as

$$(1 - y)(b_1 + b_2z + b_3x) = g.$$

Obviously, this modification will have no essential influence on the final result.

Solving the third equation for z we obtain

$$z = 1 - \frac{h}{c_1 + c_2x + c_3y}.$$

Substituting this expression for z in the first equation and performing elementary reductions, we find

$$\alpha y^2 + \beta y + \gamma = 0 \tag{1}$$

where

$$\alpha \equiv a_2c_3(1 - x),$$

$$\beta \equiv [c_3(a_1 + a_3) + a_2c_1 - c_3d] - [c_3(a_1 + a_3) + a_2(c_1 - c_2)]x - a_2c_2x^2,$$

$$\gamma \equiv [c_1(a_1 + a_3) - c_1d - a_3h] - [(c_1 - c_2)(a_1 + a_3) + c_2d - a_3h]x - c_2(a_1 + a_3)x^2.$$

The second equation (modified), when treated in the same manner, gives

$$Ay^2 + By + C = 0 \tag{2}$$

where

$$A \equiv c_3[(b_1 + b_2) + b_3x],$$

$$B \equiv [(c_1 - c_3)(b_1 + b_2) + c_3g - b_2h] + [c_2(b_1 + b_2) + b_3(c_1 - c_3)]x + b_3c_2x^2,$$

$$C \equiv - \{[c_1(b_1 + b_2) - c_1g - b_2h] + [c_2(b_1 + b_2) + b_3c_1 - c_2g]x + b_3c_2x^2\}.$$

Eliminating y from equations (1) and (2) we get

$$(\alpha C - \gamma A)^2 - (\alpha B - \beta A)(\beta C - \gamma B) = 0. \tag{3}$$

When the expressions for α, β, γ, A, B, C are substituted in relation (3) it is found that the coefficient of x^6 vanishes identically, but that all the remaining coefficients have extremely complicated values other than zero, in general. Therefore, since the equation in x is of the fifth degree with literal coefficients, the original set of equations cannot be "solved."

In the particular case where $a_1 = 2$, $a_2 = 3$, $a_3 = 4$, $b_1 = 5$, (new) $b_2 = 6$, (new) $b_3 = 7$, $c_1 = 8$, $c_2 = 9$, $c_3 = 10$, $d = 11$, $g = 12$, and $h = 13$, I found

$$1{,}628{,}991x^5 + 3{,}399{,}760x^4 - 3{,}151{,}759x^3 - 8{,}498{,}412x^2 - 8{,}179{,}912x - 8{,}133{,}008 = 0,$$

which has at least one positive real root.

411 (Algebra) [April, 1914, June, 1919]. Proposed by V. M. SPUNAR, Chicago, Ill.

Determine $x_1, x_2, x_3, \cdots x_p$, from the equations:

$$\begin{aligned} x_1 + x_2 + x_3 + \cdots + x_p &= a_0, \\ b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_px_p &= a_1, \\ b_1^2x_1 + b_2^2x_2 + b_3^2x_3 + \cdots + b_p^2x_p &= a_2, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \quad \cdot \quad \cdot \quad \cdot \\ b_1^{p-1}x_1 + b_2^{p-1}x_2 + b_3^{p-1}x_3 + \cdots + b_p^{p-1}x_p &= a_{p-1}. \end{aligned}$$

Solution by H. S. Uhler, Yale University.

From the theory of determinants, we know that

$$x_i = \begin{vmatrix} a_0 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ a_1 & b_1 & \cdots & b_{i-1} & b_{i+1} & \cdots & b_p \\ a_2 & b_1^2 & \cdots & b_{i-1}^2 & b_{i+1}^2 & \cdots & b_p^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{p-1} & b_1^{p-1} & \cdots & b_{i-1}^{p-1} & b_{i+1}^{p-1} & \cdots & b_p^{p-1} \end{vmatrix} \div \begin{vmatrix} 1 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ b_i & b_1 & \cdots & b_{i-1} & b_{i+1} & \cdots & b_p \\ b_i^2 & b_1^2 & \cdots & b_{i-1}^2 & b_{i+1}^2 & \cdots & b_p^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_i^{p-1} & b_1^{p-1} & \cdots & b_{i-2}^{p-1} & b_{i+1}^{p-1} & \cdots & b_p^{p-1} \end{vmatrix} \equiv \frac{D_1}{D_2}$$

Consider the determinant in the denominator, D_2. If any b were equal to any one of the $p - 1$ remaining b's, two columns would be identical and D_2 would vanish. Hence,